学生証番号：＿＿＿＿＿＿＿＿　氏名：＿＿＿＿＿＿＿＿＿＿

1 行列 P, Q, R, S を次のようにおく．これらの組み合わせのうち，積が定義できる場合すべてについて，その積を計算せよ．

$$P = \begin{pmatrix} -2 & 1 & -1 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 1 \\ 2 & -1 \\ -1 & 2 \end{pmatrix}, \quad R = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \quad S = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

2 Initially, three firms A, B, and C (numbered 1, 2, and 3) share the market for a certain commodity. Firm A has 20% of the market, B has 60%, and C has 20%. In the course of the next year, the following changes occur:

- A keeps 85% of its customers, while losing 5% to B, and 10% to C
- B keeps 55% of its customers, while losing 10% to A, and 35% to C
- C keeps 85% of its customers, while losing 10% to A, and 5% to B

We can represent market shares of the three firms by means of a *market share vector*, defined as a column vectors $\vec{s}$ whose components are all nonnegative and sum to 1. Define the matrix T and the initial market share vector $\vec{s}$ by

$$T = \begin{pmatrix} 0.85 & 0.10 & 0.10 \\ 0.05 & 0.55 & 0.05 \\ 0.10 & 0.35 & 0.85 \end{pmatrix} \quad \text{and} \quad \vec{s} = \begin{pmatrix} 0.2 \\ 0.6 \\ 0.2 \end{pmatrix}$$

Notice that t_{ij} is the percentage of j's customers who become i's coustomers in the next period. So, T is called the *transition matix*.

a) Compute the vectot $T\vec{s}$.

b) Show that it is also a market share vector.

c) What is the interpretation of $T(T\vec{s})$, $T(T(T\vec{s}))$, ...?

【裏に続く】

3 a) $A = \begin{pmatrix} 1 & 1 & -3 \\ 1 & -3 & 1 \\ -3 & 1 & 1 \end{pmatrix}$ の逆行列 A^{-1} を求めよ.

b) $A^{-1}A$, AA^{-1} がともに単位行列となることを確かめよ.

c) 次の連立一次方程式の解を a) の結果を用いて求めよ.

$$\begin{cases} x + y - 3z = 3 \\ x - 3y + z = -4 \\ -3x + y + z = 1 \end{cases}$$